

GALIT SHMUELI

PRACTICAL  
ACCEPTANCE  
SAMPLING

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# *Preface*

The purpose of this textbook is to introduce the reader to acceptance sampling, also called sampling inspection. While the focus of textbooks has shifted from acceptance sampling to process monitoring and control charts, acceptance sampling is still widely applied across a wide range of industries. The use of ISO or ANSI/ASQC acceptance sampling standards is common and frequently a requirement, and therefore many manufacturers, suppliers, and purchasers employ such methods. In higher education, acceptance sampling is often taught as a module in a course on quality control or industrial statistics. In other cases, it is offered as a separate course in a quality control program. This book is aimed at providing a readable and practical textbook in such courses. It is also a useful introduction for professionals who strive to better understand acceptance sampling. From our experience, learning is best achieved by doing. Hence, we designed the book to achieve self-learning in the following ways:

- The book is relatively short compared to other books on acceptance sampling, to reduce reading time and increase hands-on time.
- Explanations strive to be clear and straightforward with more emphasis on concepts than on theory.
- Each chapter has end-of-chapter problems, ranging in focus, with several requiring computations using basic software such as Excel or online calculators.

The book is designed for a 5-6 week module that is suitable for graduate or upper-undergraduate level. A pre-requisite is a

basic course in statistics and probability that covers distributions such as the Binomial, Hypergeometric and normal distributions, the concept of expected values, and summary statistics. A suggested schedule is:

*Weeks 1-2* Chapters 1 and 2 (Introduction, terminology, and single-stage plans for attributes). It is advisable to include a refresher on the Binomial, Hypergeometric and normal distributions.

*Week 3* Chapters 3-4 (double-stage sampling plans, rectifying inspection plans).

*Week 4* Chapter 5 (Plans for variables (measurement data)).

*Week 5* Chapter 6 (Continuous sampling plans)

*Week 6* Project presentations or final exam.

*We strongly recommend including a team project in such a course, where students work on a real problem where acceptance sampling can be beneficial.*

In terms of software, Microsoft Excel and online calculators are used throughout the book to illustrate the different methods and procedures. The reason for choosing Excel is due to its wide accessibility in industry and the prior familiarity of most students with it. We use functions in Microsoft Excel 2010 which are also available in earlier versions of Excel (2003 and 2007), although function names might be slightly different. We also use Excel for creating charts, although careful manual formatting is necessary for producing effective graphs.

### *What's new in the second edition?*

The second edition provides several improvements, based on feedback from readers and on software updates. The book now includes a section on Accept-on-Zero plans. Additional screenshots from the newly-designed SQCOnline.com illustrate several

new calculators. Finally, the second edition offers an improved design for enhanced readability.

Supporting materials for this book are available at [www.galitshmueli.com/practical-acceptance-sampling](http://www.galitshmueli.com/practical-acceptance-sampling).

# 1 Introduction

## 1.1 Acceptance Sampling in Industry and Commerce

In today's manufacturing world, the path from raw materials to final product often takes place over multiple companies and across multiple continents. A company selling laptop computers has likely sub-contracted different parts to different sub-contractors. The assembly might take place in yet another plant. And the packing and shipping, from yet another location. In order to assure a certain quality level, companies use inspection at the different supply chain stages.

*Acceptance Sampling*, also known as *Sampling Inspection*, consists of quality assurance schemes designed to test whether the quality of batches of products or services conform with requirements, based on inspecting *only a sample* from each batch. The use of sampling inspection relies on the premise that products need not conform 100% with specification requirements, and it is often more economical to allow a small percentage of non-conforming items to pass on for later rejection than to bear the expense of 100% inspection. Acceptance sampling provides criteria and decision rules for determining whether to accept or reject a batch based on a sample. Civilian ISO acceptance sampling plans and their military counterparts (Mil-Std) are commonly used standards in industry. These plans dictate the sample size to be drawn from each batch, and the requirements that the sample must meet to assure that the entire batch is of acceptable quality.

Acceptance sampling is useful for testing the quality of batches of items, especially when a large number of items must

be processed in a short time. However, acceptance sampling is not a method for monitoring or improving the quality of a process. It is important to keep this in mind and to use other quality control tools such as control charts for monitoring and designed experiments for improvement.

The origin of Acceptance Sampling is in World War II, for testing the quality of bullets in the United States' military. The need for sampling arose from the destructible nature of bullet testing. In general, acceptance sampling is needed either when inspection is destructive, or when inspecting each item (100% inspection) is prohibitively expensive or time-consuming.

Acceptance sampling is commonly used in contracting and sub-contracting, where the contractor wants to assure the quality of the incoming goods or services. For example, a school providing lunch services inspects the food quality on a daily basis. A large cosmetics company inspects the quality of the products in each shipment that it receives from its overseas manufacturers. A large mining manufacturing company in the USA uses acceptance sampling to inspect machined parts from its vendors. The famous coffee chain Starbucks uses acceptance sampling to assure the quality of coffee shipments from its different suppliers.

Because of Starbucks' high volume, coffee exporters competed to become suppliers. For its part, Starbucks cultivated long-term relationships with its suppliers, providing training working closely with them. For quality control, Starbucks extracted three different samples from every shipment: one before the export was arranged, one just before shipment and one on arrival at the roasting plant. At each stage Starbucks reserved the right to refuse the shipment<sup>1</sup>.

Acceptance sampling is also commonly used by manufacturers or suppliers themselves, as a tool for audit or compliance. A well-known vendor of child safety car seats inspects the quality of its products using acceptance sampling. A call center audits a sample of calls in each shift to assure quality service. A manufacturer and exporter of home textiles in India employs inspection after the fabric cutting stage, after sewing, and after packing. Manufacturers of electronic parts in China use sampling inspection to assure that shipments of their parts will be

<sup>1</sup> from *The Starbucks Brand* case study, Rotman School of Management, University of Toronto, [www.rotman.utoronto.ca/bic/caseseries/PDFs/starbucks.pdf](http://www.rotman.utoronto.ca/bic/caseseries/PDFs/starbucks.pdf), accessed Aug 1, 2011.



up to the standard of their purchasing companies.

### 1.2 *Who Should Understand Acceptance Sampling?*

Anyone involved in the inspection phase should be literate of what acceptance sampling is, how to deploy it effectively, and what sampling plans guarantee. This includes management, R&D, and quality personnel. In particular, production and process engineers, manufacturing and design engineers, purchasing agents and managers, and quality engineers, inspectors, supervisors, and managers.

### 1.3 *What Are Sampling Plans?*

Acceptance Sampling is based on the notion of *probability*: Although we cannot deduce from a sample about the exact quality of the entire batch, we can reach conclusions with a given certainty level. For example, using a sampling plan and examining a sample, we might conclude that "we are 95% certain that the percent of non-conforming items in the entire batch is no more than 1%".

Sampling plans consist of *criteria*, such as the size of the sample to inspect and *decision rules*, which tell us whether to accept or reject the batch, based on the quality of the inspected sample.

Samples are typically *drawn at random*. Drawing a random sample is sometimes operationally costly or inconvenient, for instance, when a shipment consists of many boxes that are piled one on top of the other. However, random sampling is extremely important for drawing correct decisions. Although it might be easier to sample all items from the top-most box, the sample information might be biased if the top-most box is slightly different in quality from all the other boxes in the shipment.

Sampling plans are divided into two general categories, depending on the quality property that is being measured:

*Sampling plans for attributes* are used for pass/fail (go/no-go) classifications such as whether a phone call disconnects, and for the number of non-conformities, such as the number of

disconnected calls per hour.

*Sampling plans for variables* are used for continuous measurements such as the length of a phone call. Sampling plans can also range from single-stage plans, where a single sample is drawn from a batch, to double-stage plans, where one or two samples are taken, to multiple-stage plans, where one or more samples are drawn.

#### 1.4 Terminology

*Lot, batch, and shipment* are used interchangeably to denote the collection of items to which an accept/reject decision must be made. The size of a batch is denoted by  $N$ .

*Sample* refers to a subset of a batch that is typically drawn at random, and is fully inspected. Sample size is denoted by  $n$ .

*Producer* refers to the party who manufactures or creates the good or service of interest. The producer is assumed to produce items on a regular basis.

*Consumer* refers to the party who purchases goods or services from the producer. The consumer receives batches of goods or services from the producer.

We use  $X$  to denote the random variable measuring the quantity of interest in the sample. For pass/fail data,  $X$  measures the number of failures in the sample. For example, if a sample of phone calls contains 3 disconnected calls, we denote this by  $X=3$ . For continuous measurements we use  $X_i$  to denote the measurement of the  $i$ th item in the sample. For example, if the first milk carton in a sample of 10 cartons contains 500cc, we use the notation  $X_1=500$ .

$p$  is used to denote the proportion of non-conforming ("failed") items in the batch.

## 1.5 What determines a sampling plan?

### *Producer's Side*

A producer produces items on a continuous basis (think of a production line). Let us assume that the production line quality meets a baseline requirement called *Acceptable Quality Level* (AQL). The producer would like to assure that batches from this process will be accepted with a high probability.

The producer's risk (denoted  $\alpha$ ) is the chance that a batch from a process with quality AQL is rejected.

### *Consumer's Side*

The consumer cares about the quality of individual batches. The consumer's required per-batch quality is called *Lot Tolerance Percent Defective* (LTPD). The consumer would like to reject batches with quality LTPD or worse with high probability.

The consumer's risk (denoted  $\beta$ ) is the chance that a batch with quality LTPD or worse is accepted.

A *sampling plan* is a scheme aimed at balancing the requirements of the producer and the consumer. It does so by *balancing the producer's risk ( $\alpha$ ) and the consumer's risk ( $\beta$ )*.

We will formalize these concepts in the next sections, within the context of specific sampling plans.

## 1.6 Problems

1. What are two advantages of acceptance sampling over 100% inspection?
2. What is the main purpose of acceptance sampling?
3. *Bee Healthy* is a coop of honey collectors in India. The coop regularly inspects batches of their honey jars to assure that quality is up to the standards of an organic certifying agency. A honey jar can either be conforming or non-conforming. Each batch consists of 300 jars.

- (a) Should *Bee Healthy* use sampling plans for attributes or for variables? Explain.
- (b) Is *Bee Healthy* considered the producer or the consumer? Explain.
- (c) What is the meaning of the producer's risk in this case?
- (d) How should a sample be drawn from each batch of 300 jars?